Deconstructing Spatiotemporal Chaos Using Local Symbolic Dynamics

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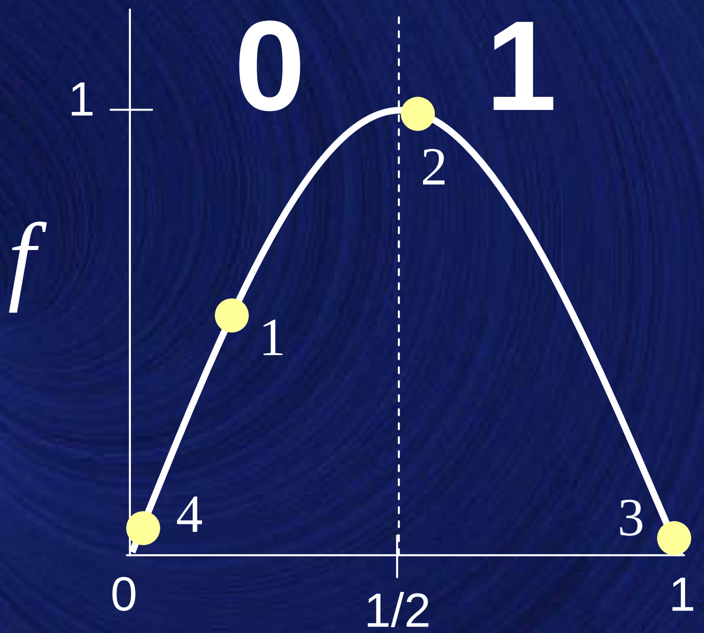
Overview

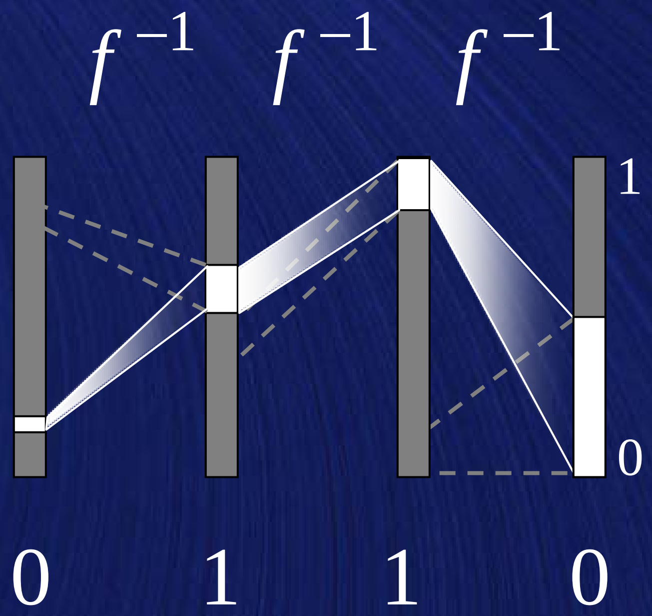
- Symbolic Dynamics
- Extension to Coupled Map Lattice
- Local Symbolic Dynamics
- Controlling Spatiotemporal Chaos
- A Flow Example

Symbolic Dynamics of Logistic Map

$$f(x) = 4x(1-x)$$

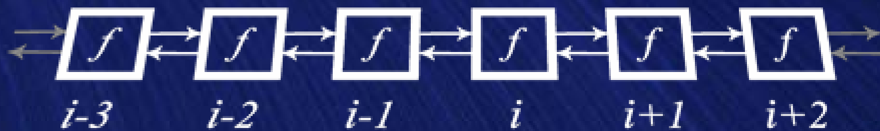


$$x = 0.16$$



Extension to Coupled Map Lattices

⋮	⋮	⋮	⋮	⋮	⋮	
0.29	0.33	0.16	0.83	0.29	0.76	t+7
0.93	0.90	0.97	0.65	0.94	0.68	t+6
0.60	0.34	0.52	0.19	0.56	0.88	t+5
0.79	0.92	0.83	0.96	0.81	0.46	t+4
0.74	0.63	0.71	0.57	0.27	0.97	t+3
0.75	0.81	0.24	0.18	0.93	0.15	t+2
0.24	0.70	0.94	0.97	0.44	0.57	t+1
0.95	0.23	0.61	0.49	0.89	0.91	t

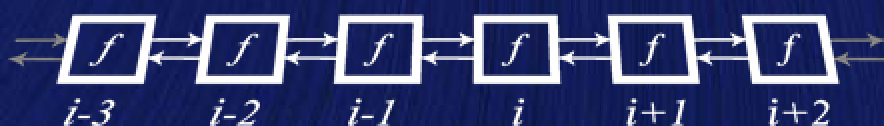


$$X_{n+1} = A \circ F(X_n)$$

$$F(X) = \begin{pmatrix} f(x^1) & \cdots & f(x^N) \end{pmatrix}$$

$$X_n = F^{-1}(A^{-1} X_{n+1})$$

⋮	⋮	⋮	⋮	⋮	⋮	
0	0	0	1	0	1	t+7
1	1	1	1	1	1	t+6
1	0	1	0	1	1	t+5
1	1	1	1	1	0	t+4
1	1	1	1	0	1	t+3
1	1	0	0	1	0	t+2
0	1	1	1	0	1	t+1
1	0	1	0	1	1	t



Local Symbolic Dynamics

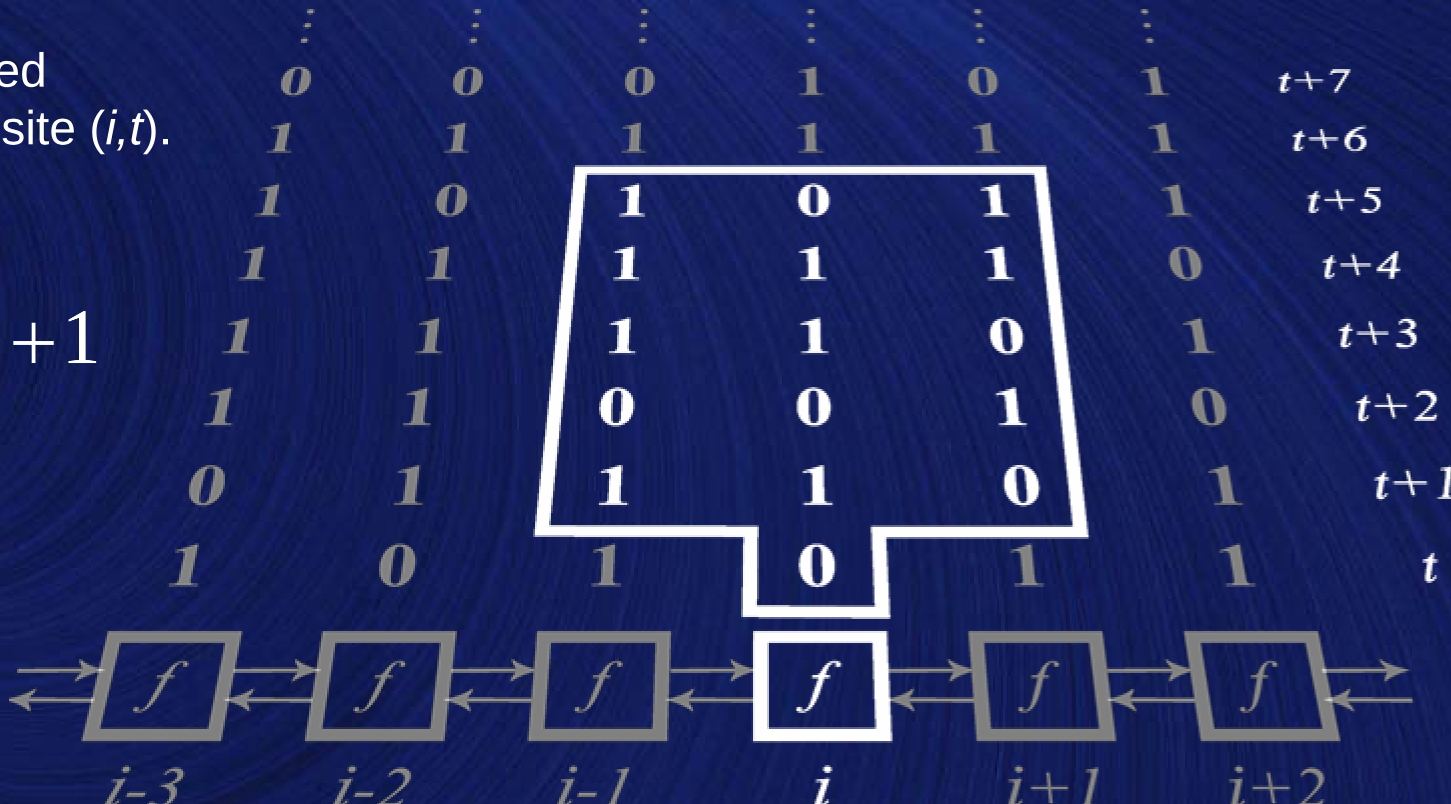
Example m = 3, n = 6

$$m(n-1)+1$$

symbols used
to describe site (*i*,*t*).

versus

$$N(n-1)+1$$



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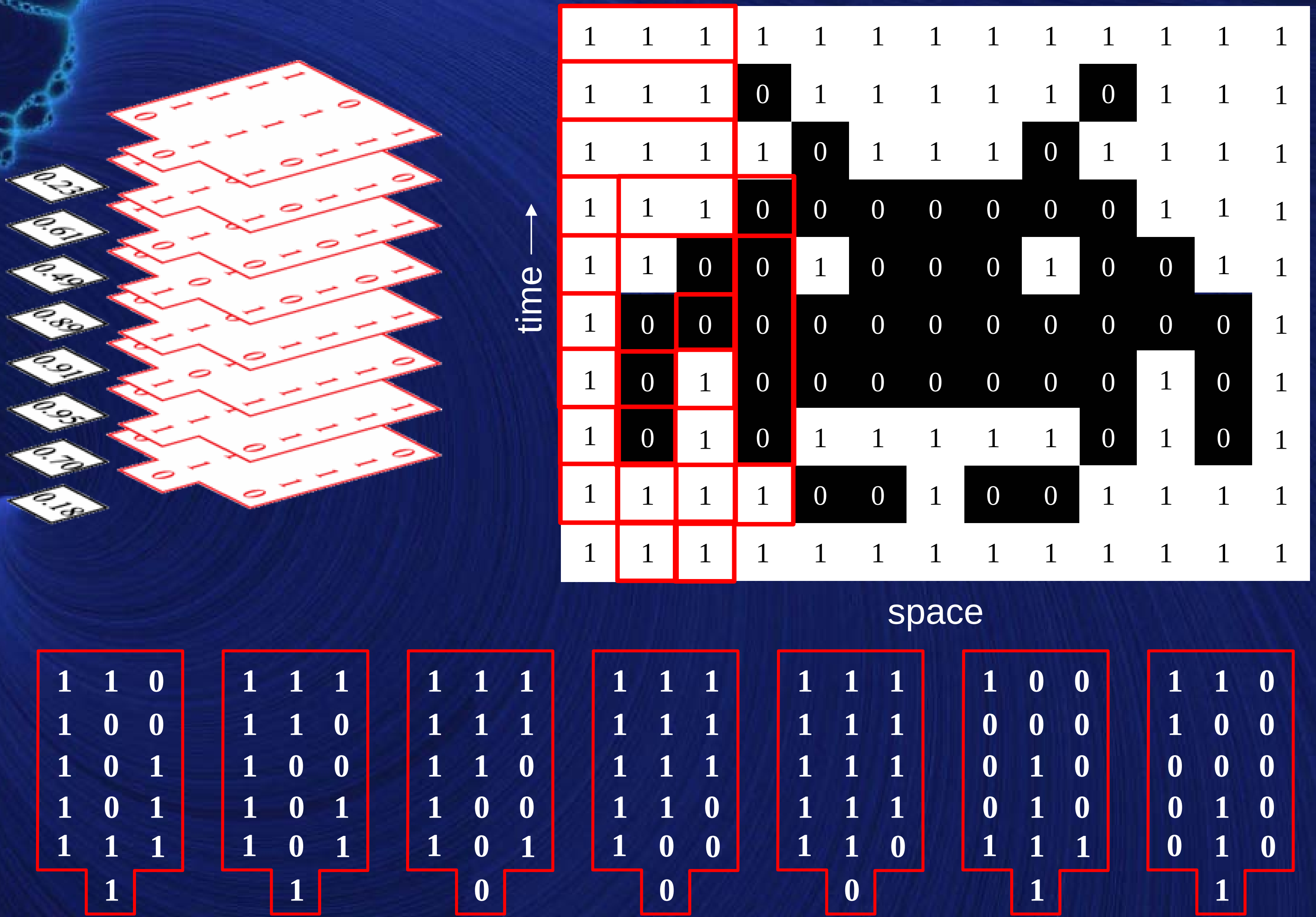
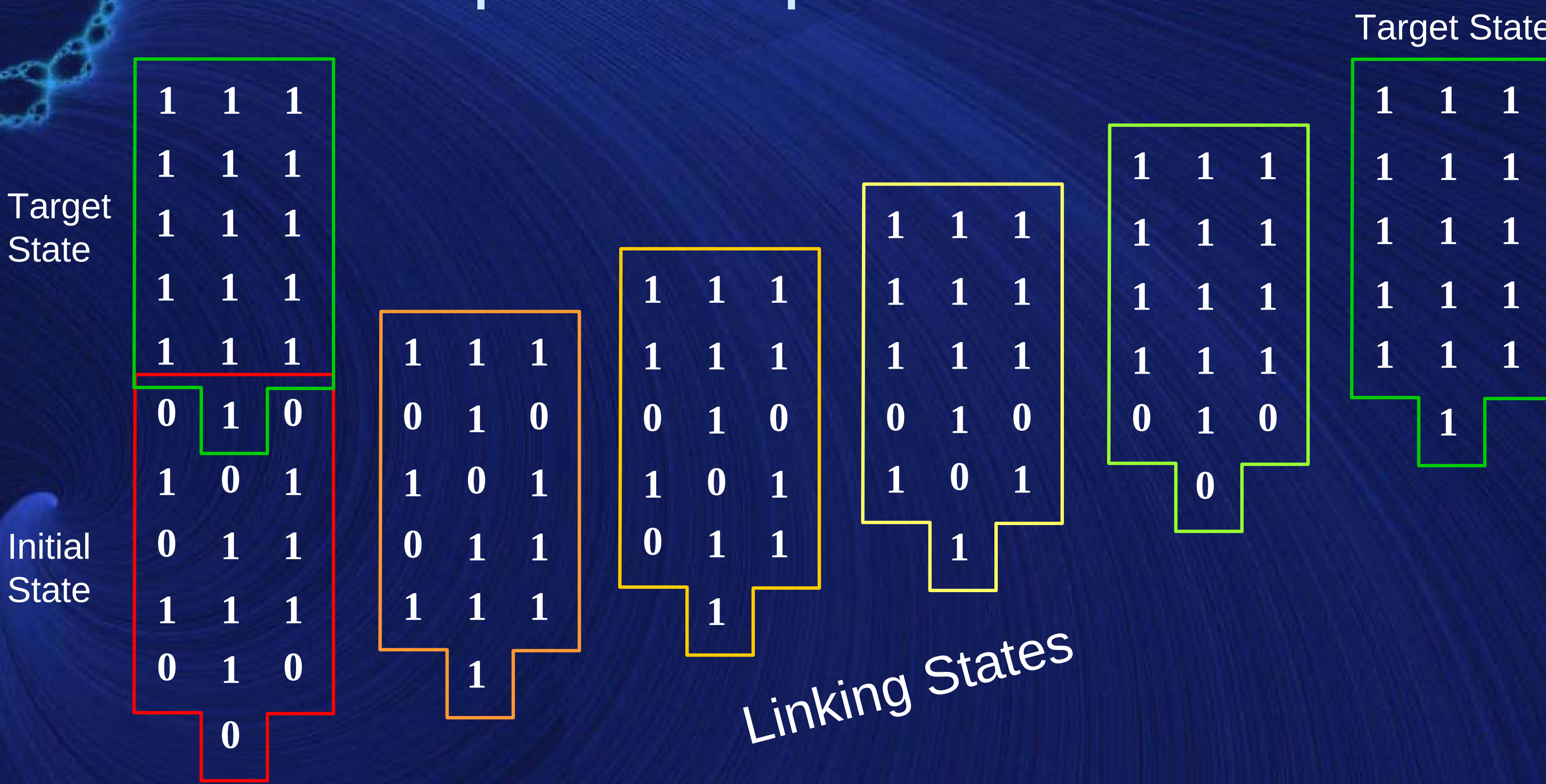
Constructing Global States



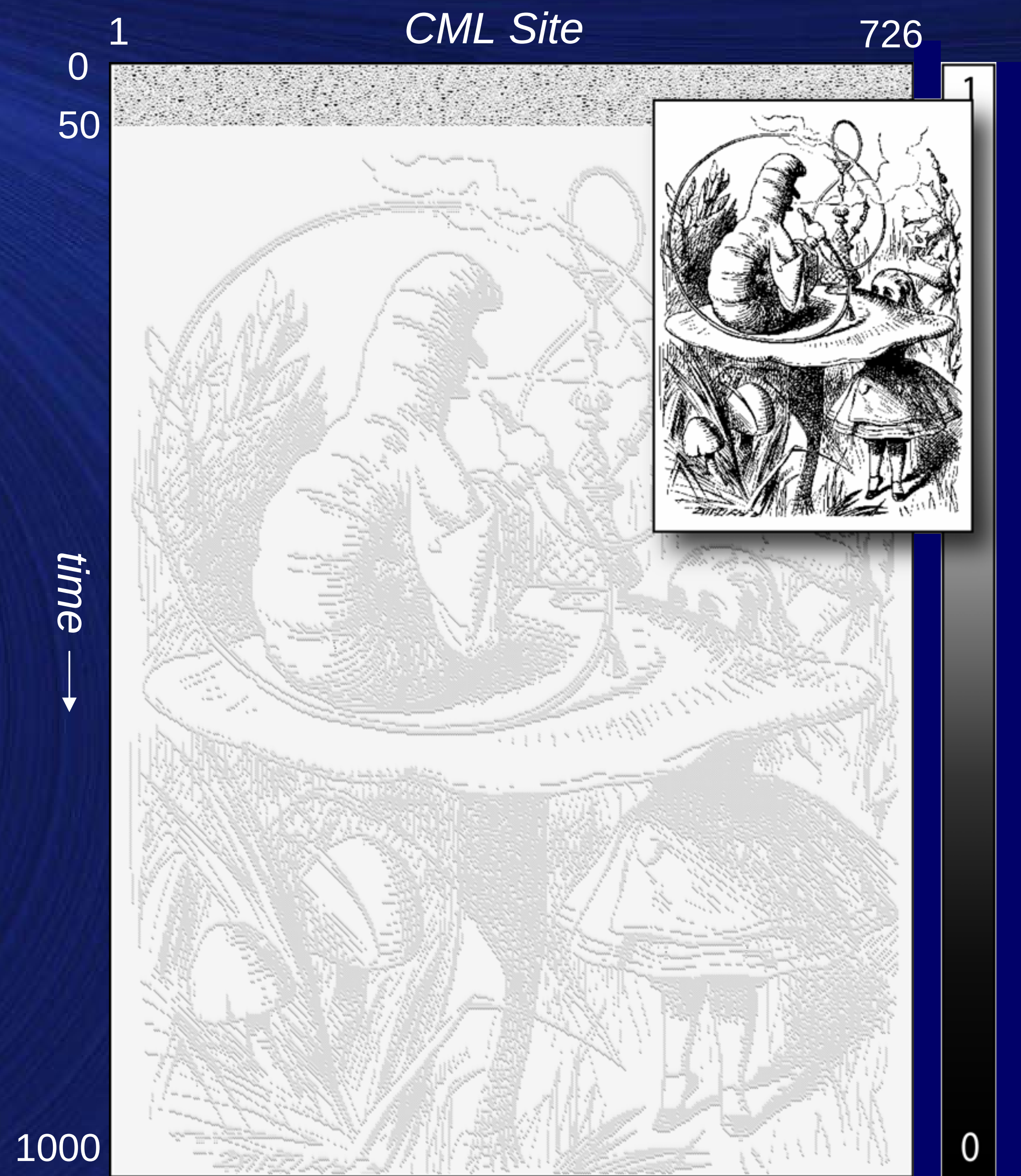
Targeting States of Spatiotemporal Chaos



Append the desired target symbol sequence onto the end of the current symbolic state. Mapping back to state space gives us the connecting orbit.

Controlling a Complex Orbit

- N=726, ε = 0.1 logistic CML
- At t = 50 the CML is steered onto target orbit. White pixels = ‘1’, black ‘0’.
- An m=3, n=6 LUT is used to find the required CML site values
- Controller pushes the CML state onto the desired orbit at each iteration
- Mean control signal is 0.007 (0.7% of the dynamic range of each element.)



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High-Dimensional Flow

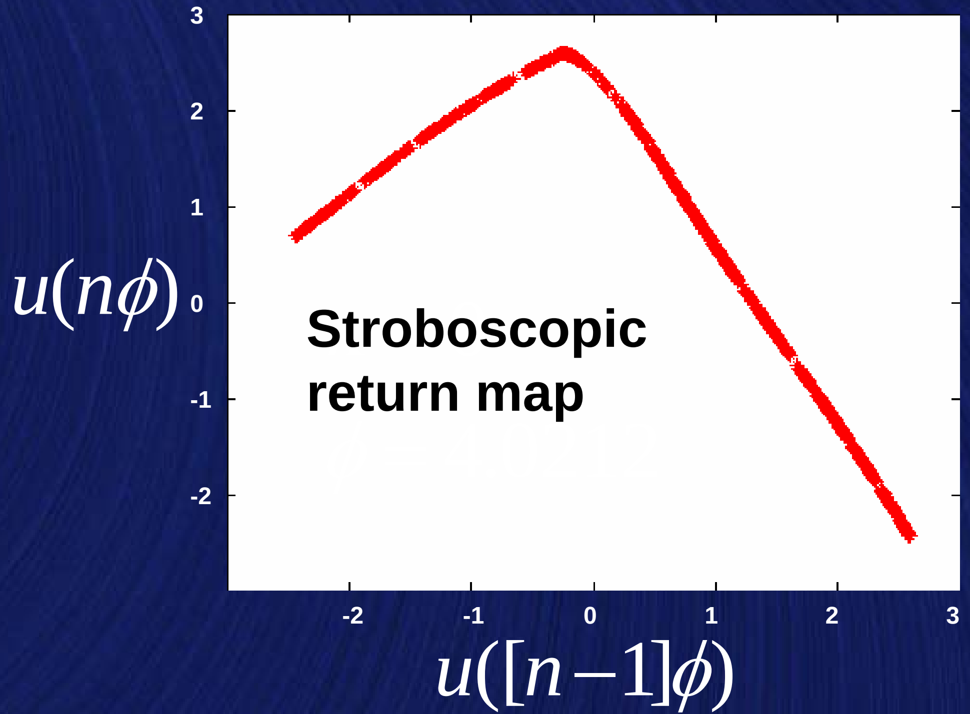
Consider the driven reaction-diffusion system:

$$\dot{u}_i = 0.5 - 4v_i + \kappa(u_{i+1} + u_{i-1} - 2u_i)$$
$$\dot{v}_i = -v_i + 2\max(u_i - 8\cos t - 16, 0)$$

"Regularized Rossler"

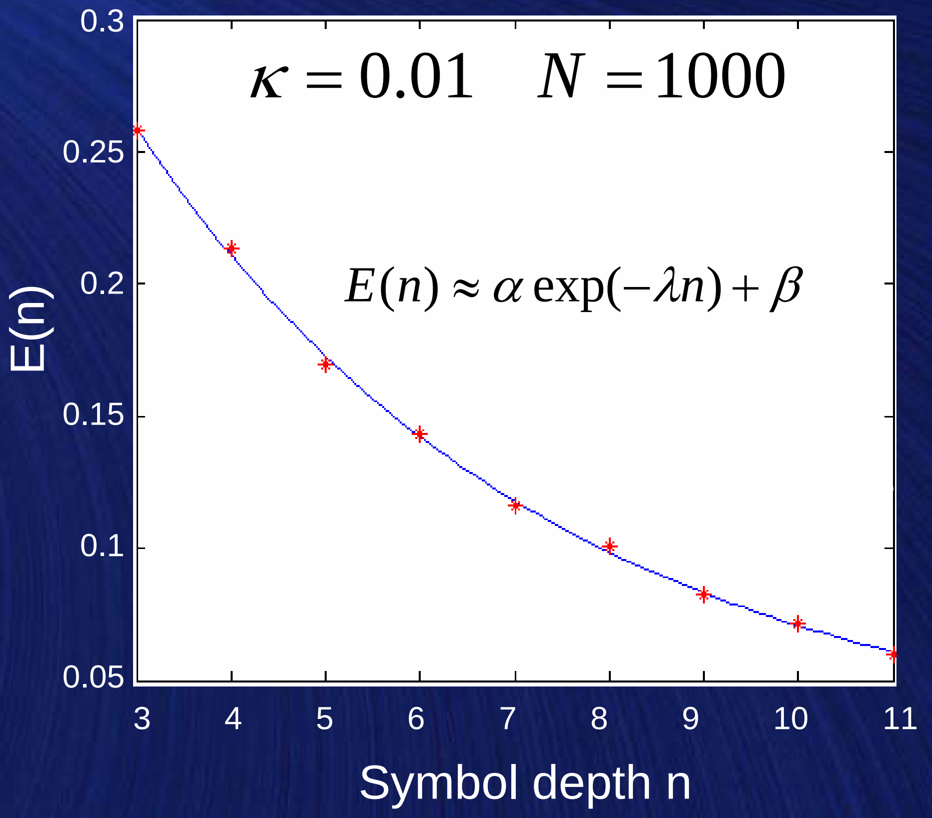
The return map for the uncoupled (k = 0) oscillator has a unimodal shape.

<u>Ansatz</u>: Return map for k >0 described by a coupled lattice of unimodal maps



High-Dimensional Flow

- 1000 returns of $u_i(t)$ collected
- We estimate $u_p = -0.12$ by finding $\sup\{ u_i(t) \}$ and taking prior iterate at that location
- Symbolize: $s_i(t) = 1$ if $u_i(t) > u_p$, 0 otherwise
- Compile m = 3 LUT, compute E(n)
- Error less than 1% at n=11



The local symbolic model performs as well for this case of ODEs as it does for the CML.

Conclusion

- Systems that can be modeled as diffusively coupled lattices of unimodal maps are likely to have a compact description in terms of local symbolic models.

- For these systems chaos control is straightforward and novel global states can be predicted and targeted based on previously measured data.

- The approach discussed here is easily generalized to multi-dimensional lattices and to 1-d maps of more than two symbols.

- We think it likely that any network of small in-degree is a good candidate for reduction to a local symbolic model.

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