A coupla ducks

Slow currents, canards, and synchrony

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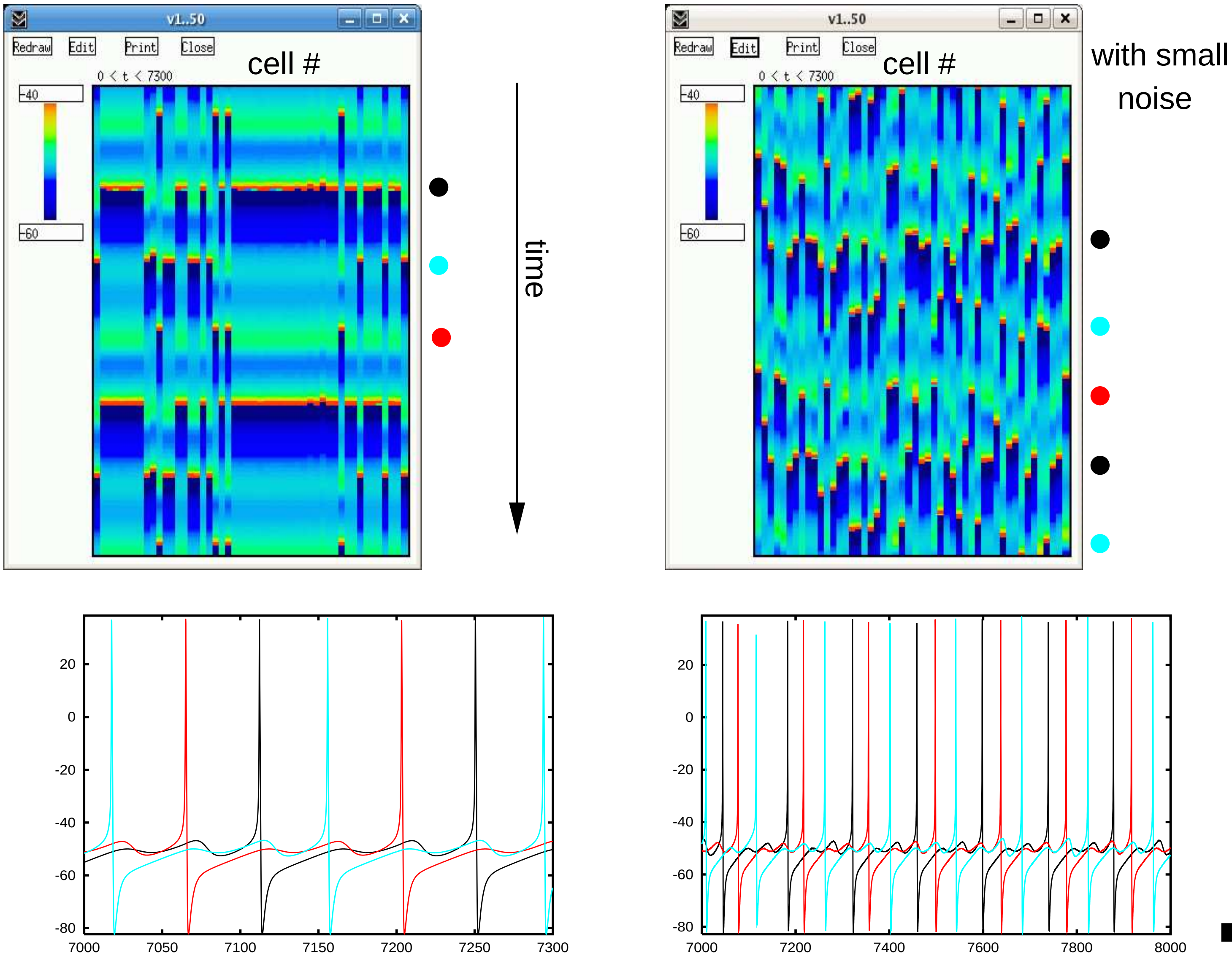
Outline

- Erisir model for inhibitory interneurons
- Simulations of a pair/ many
- Bifurcation of a single cell
- Weak coupling analysis

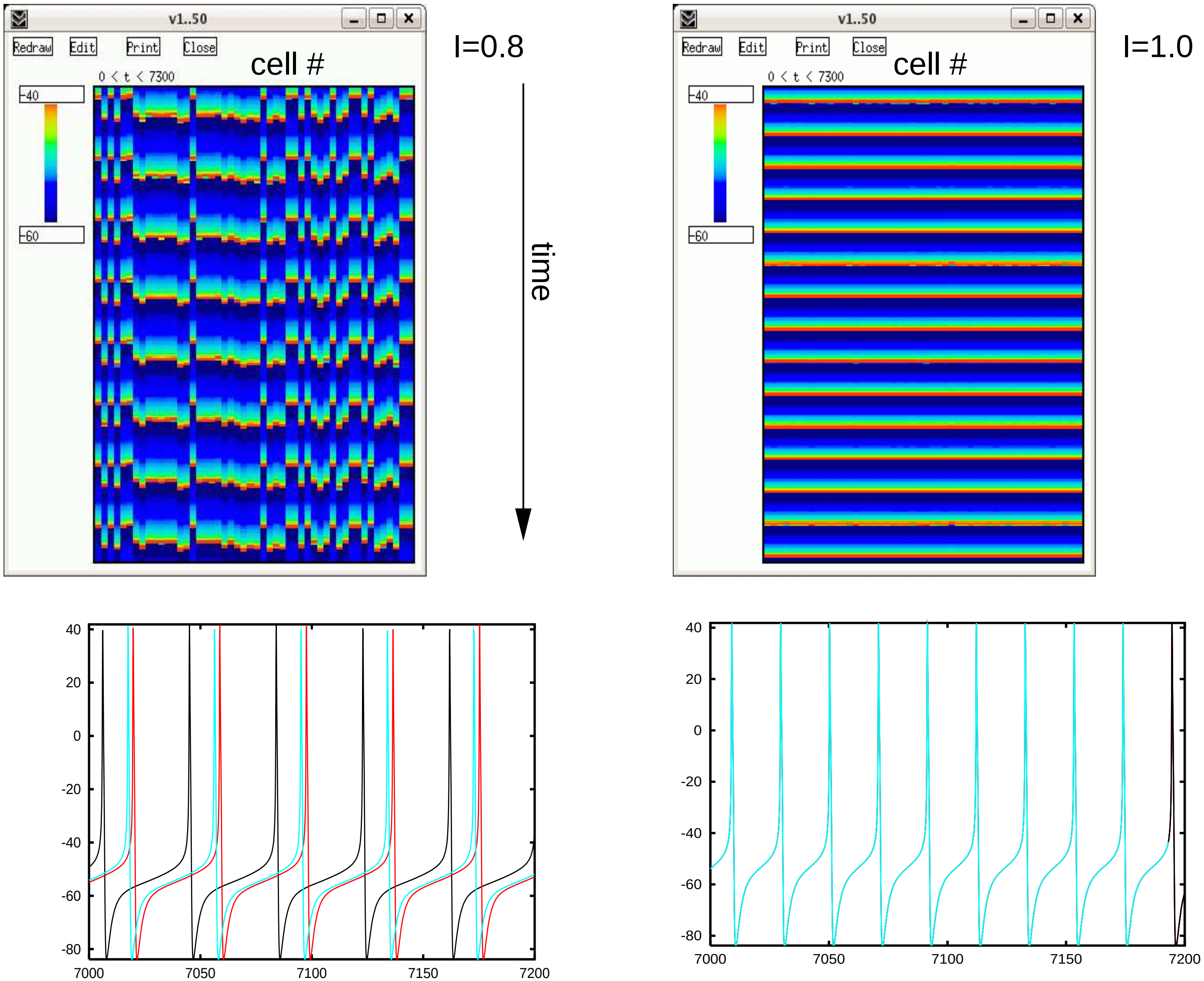
Fast spike units

- Dominant interneuron in sensory cortex
- Erisir et al (J. Neurophy. 1999)
  - Two types of potassium channels
  - Need sustained high frequency firing
- Synchrony of inhibition important
  - Mediated by fast inhibition
  - Gap junctions

GJ network of 50 all:all

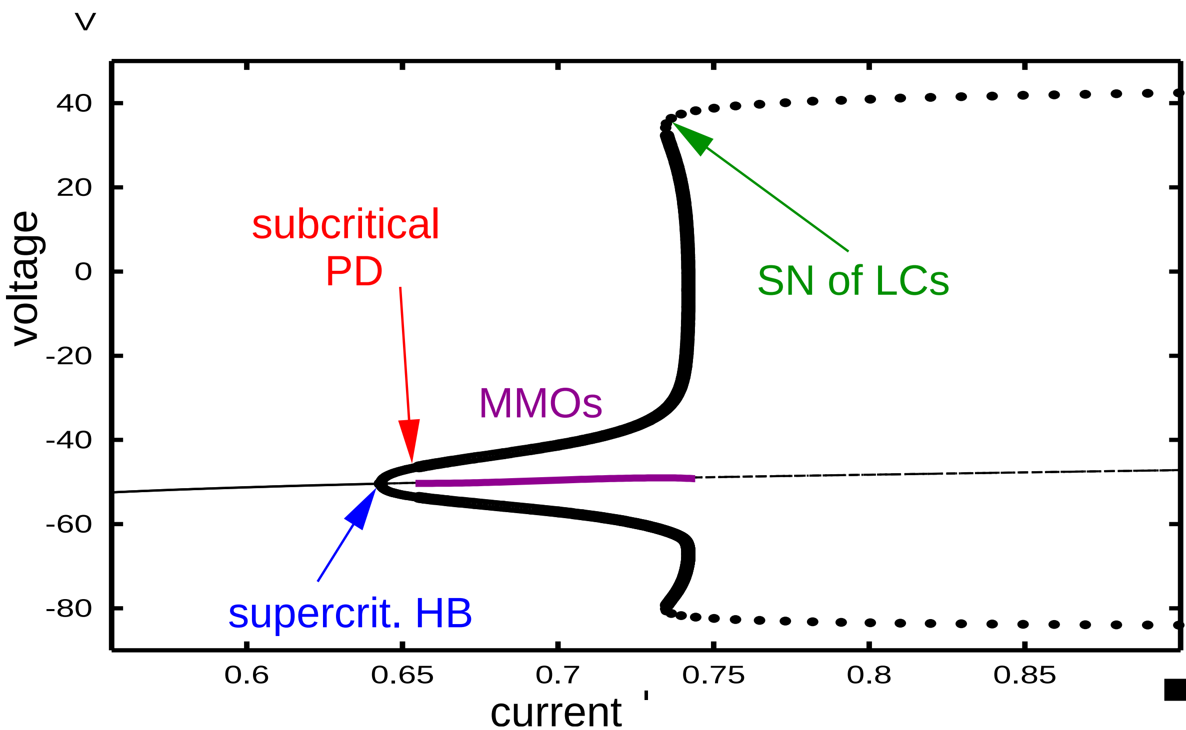


Small increase in current

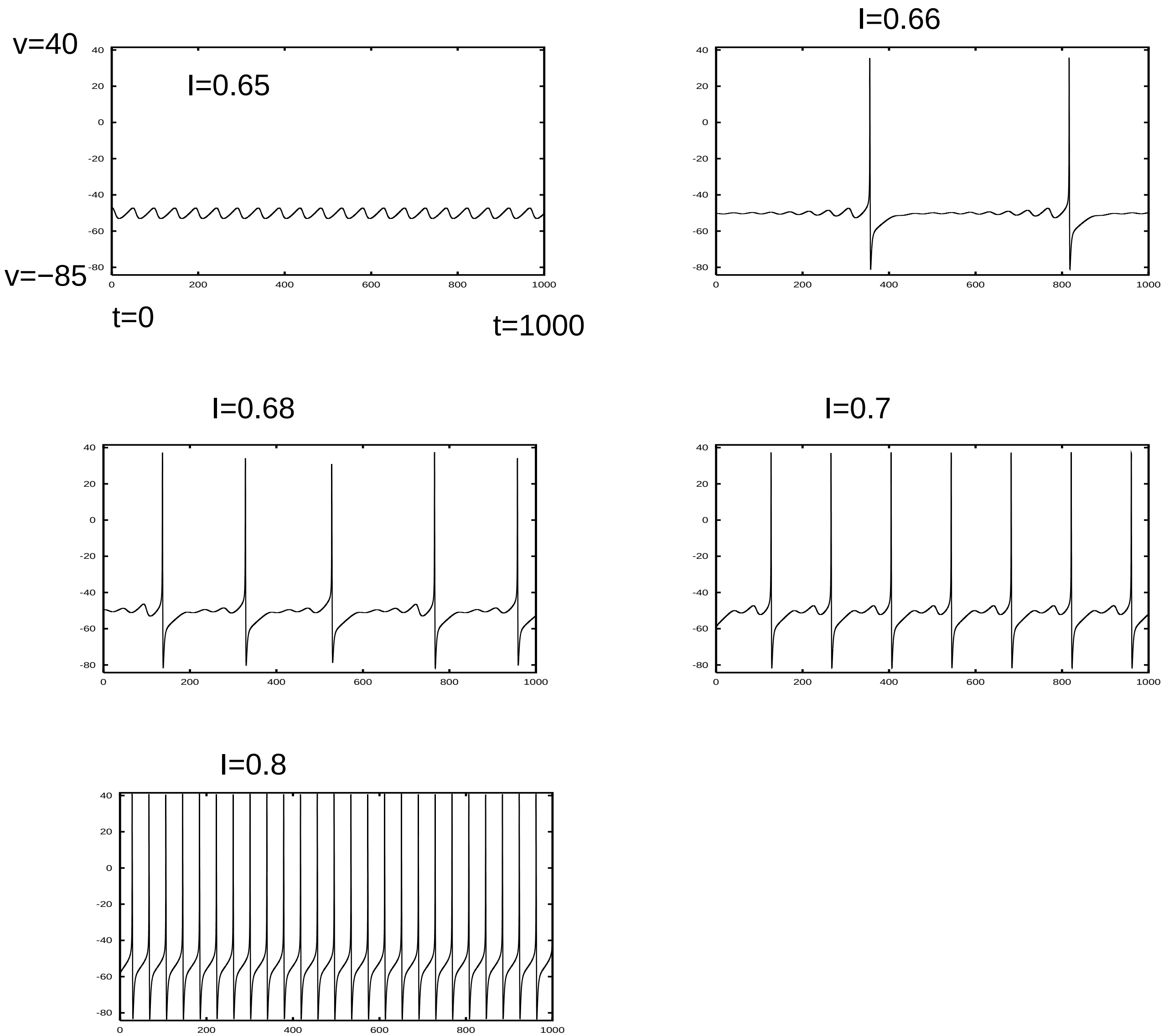


What underlies the transition?

- Recalling the voltage traces, there appear to be small subthreshold oscillations between spikes.
- Suggests, possibly, mixed-mode oscillations
- Single cell bifurcation diagram

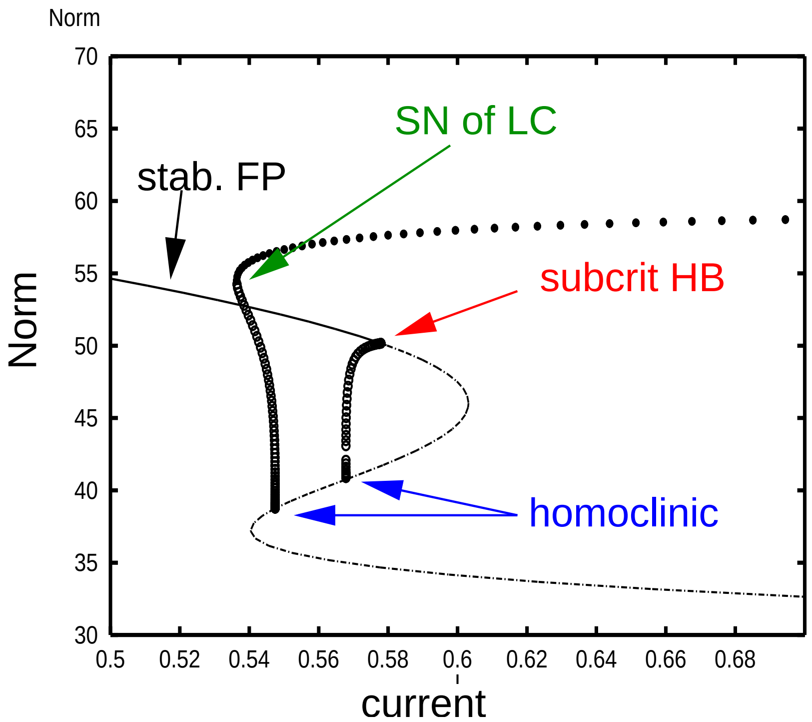


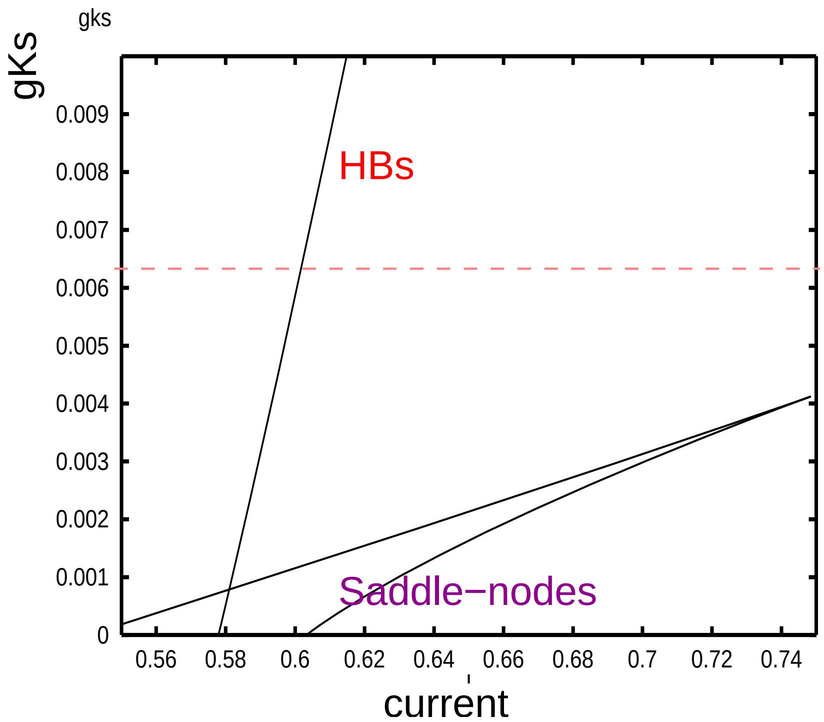
Voltage series



What underlies this behavior?

The slow potassium conductance, g_{Ks} appears to be responsible for this





Weak coupling

$$\frac{dX_j}{dt} = F(X_j) + \epsilon G_j(X_k, X_j)$$

$X_j(t) = X_0(\theta_j) + \epsilon Y_j(t)$, and

$$\frac{d\theta_j}{dt} = 1 + \epsilon H_j(\theta_k - \theta_j)$$

where

$$H(\phi) = \frac{1}{T}\int_0^T X^*(t) \cdot G_j(X_0(t+\phi), X_0(t))\, dt$$

and $X^*(t)$ is the normalized periodic solution to

$$\frac{dX^*(t)}{dt} = -[D_X F(X_0(t))]^T X^*(t).$$

The voltage component is the PRC.

Pairwise interactions

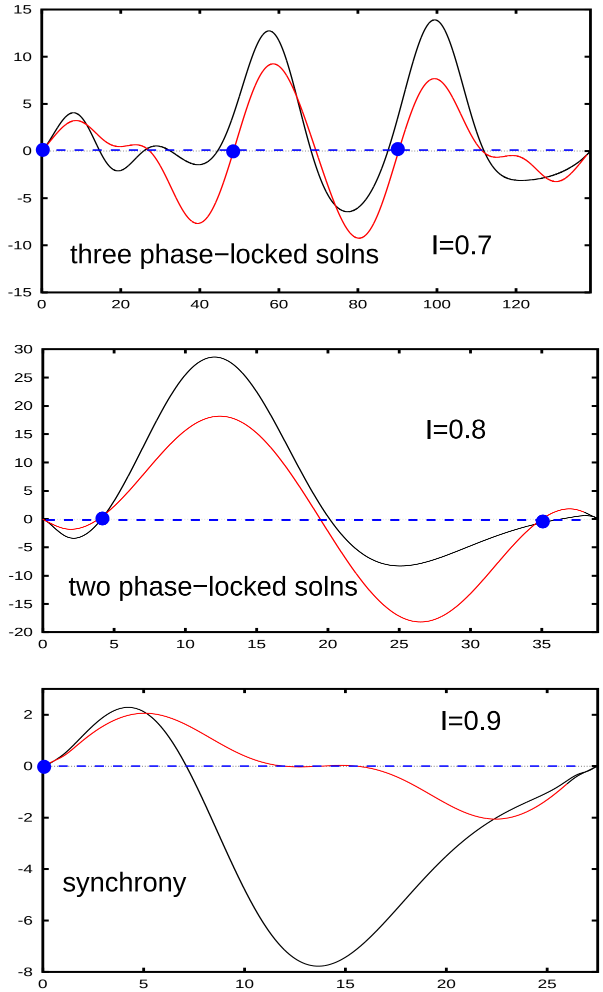
$$\begin{aligned}\frac{d\theta_1}{dt} &= 1+\epsilon H(\theta_2-\theta_1)\\ \frac{d\theta_2}{dt} &= 1+\epsilon H(\theta_1-\theta_2)\end{aligned}$$

Letting, $\phi=\theta_2-\theta_1$:

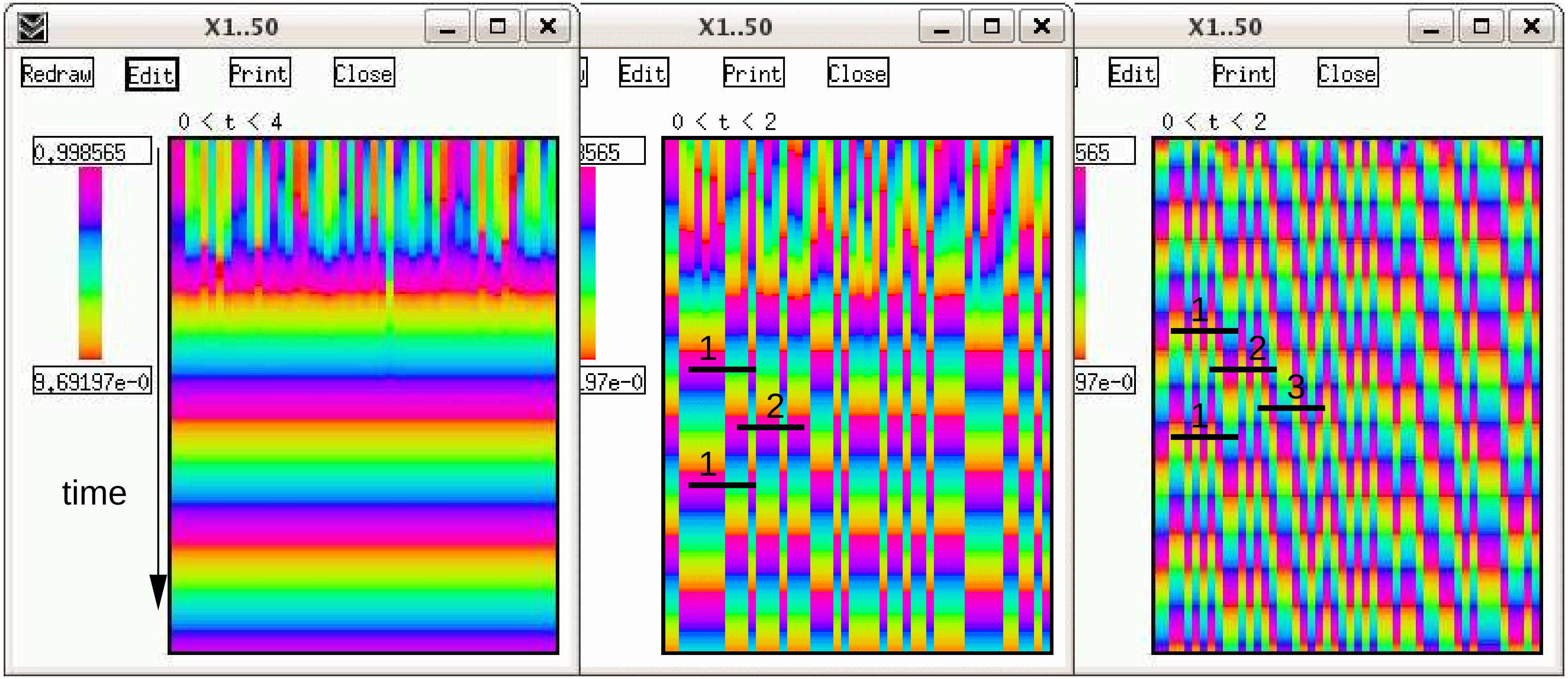
$$\frac{d\phi}{dt}=H(-\phi)-H(\phi)\equiv -2H_{odd}(\phi).$$

Zeros of $H_{odd}(\phi)$ correspond to **phase-locked** solutions to the full ODEs

Phase model for pairs



Phase models explain full model



I=0.9 I=0.8 I=0.7

Clustering

Which modes are chosen?

$$\theta_i' = \omega + \frac{1}{N} \sum_j H(\theta_j - \theta_i) + \sigma \xi_i$$

becomes:

$$\begin{aligned} \frac{\partial p(\theta,t)}{\partial t} &= -\frac{\partial J(\theta,t)}{\partial \theta} + \frac{\sigma^2}{2} \frac{\partial^2 p(\theta,t)}{\partial \theta^2} \\ J(\theta,t) &= p(\theta,t) \left[\omega + \int_0^1 H(y-\theta) p(y,t)\, dy \right]. \end{aligned}$$

Instability

Asynchronous state:

$$p(\theta, t) = \frac{1}{2\pi}$$

loses stability as the noise (σ^2) decreases. Dominant mode is determined from

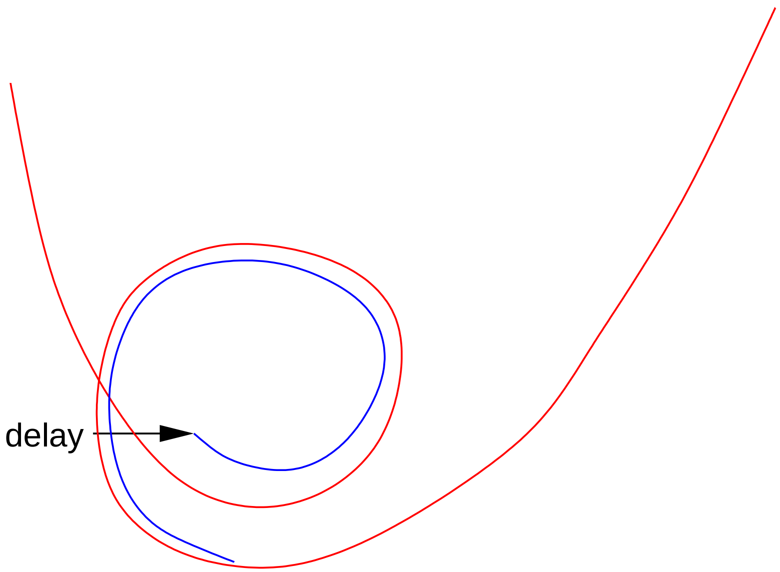
$$\frac{\sigma_{crit}^2}{2} = \max_n \{\frac{a_n}{n}\}$$

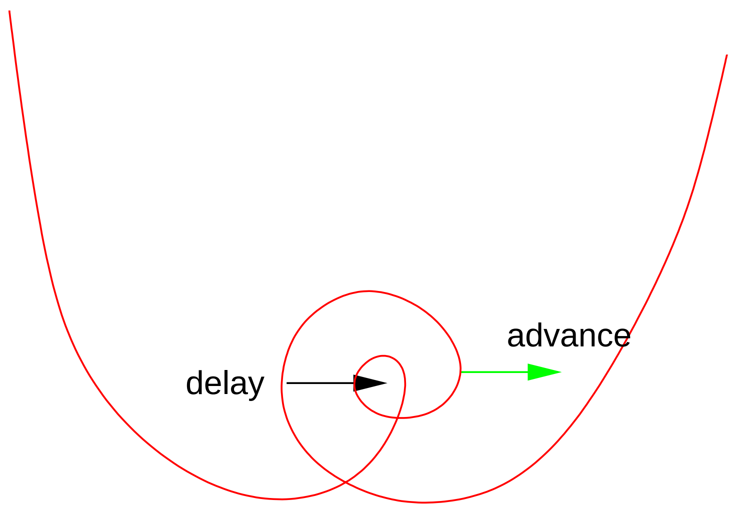
where a_n are sine components of H.

For $I = 0.9, n = 1$, for $I = 0.8, n = 2$, and for $I = 0.7, n = 3$.

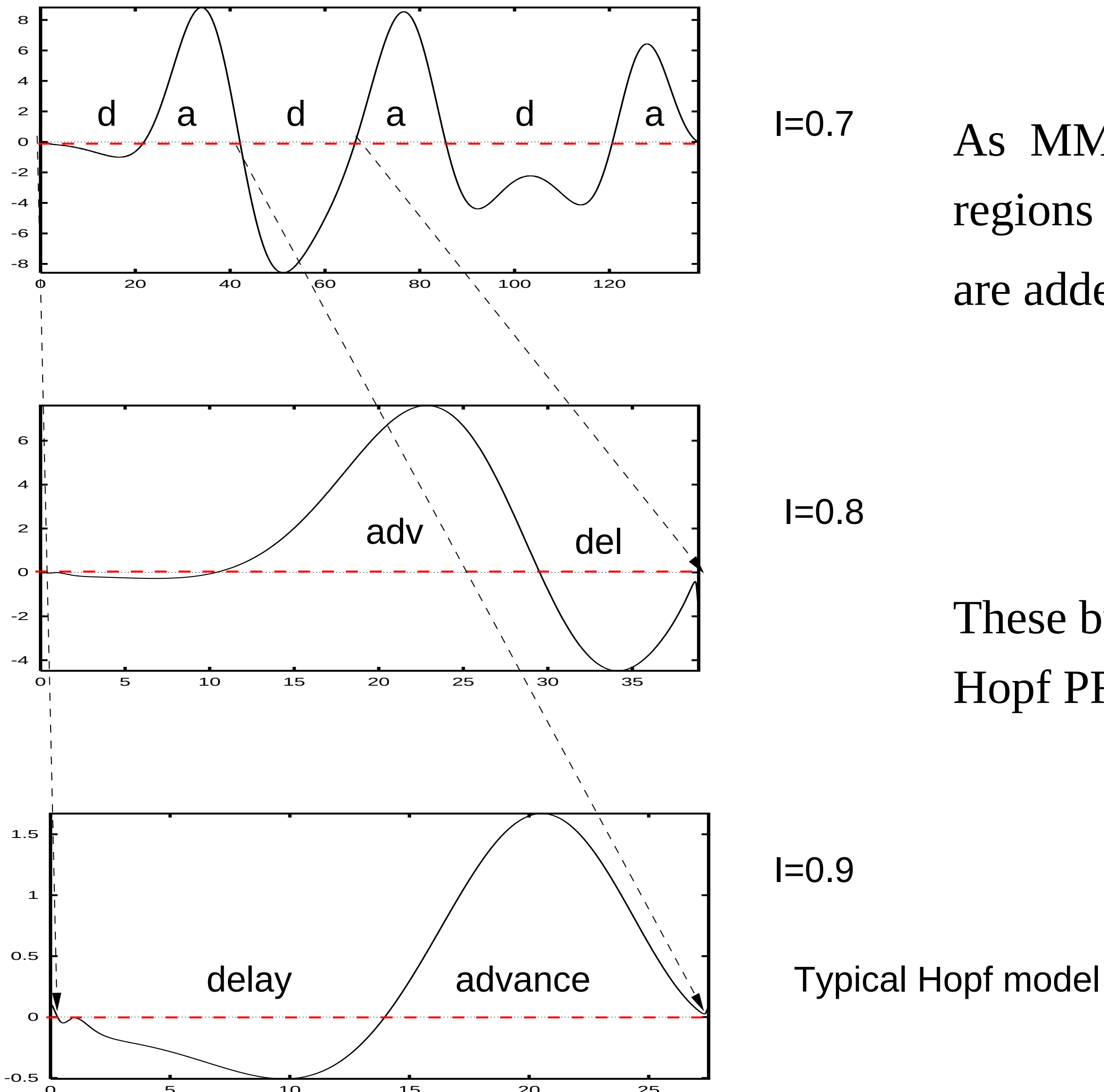
What accounts for the mode shift?

The “mixed modes” allow for multiple times at which the spike can be advanced and delays. This results in the phase-resetting curve having high frequency components.





PRCs for MMOs

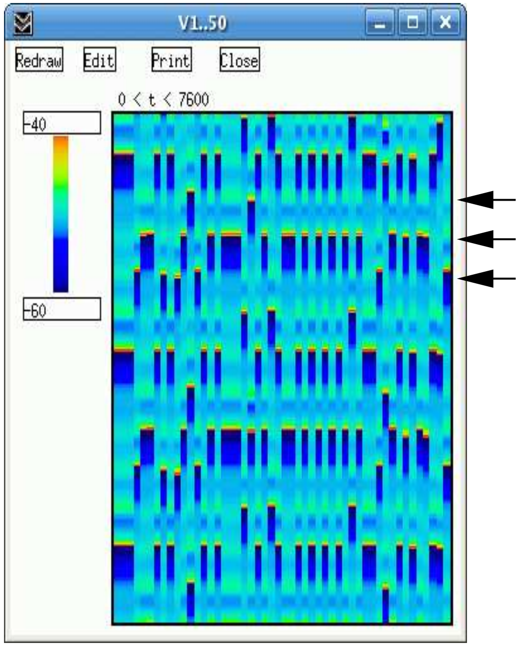


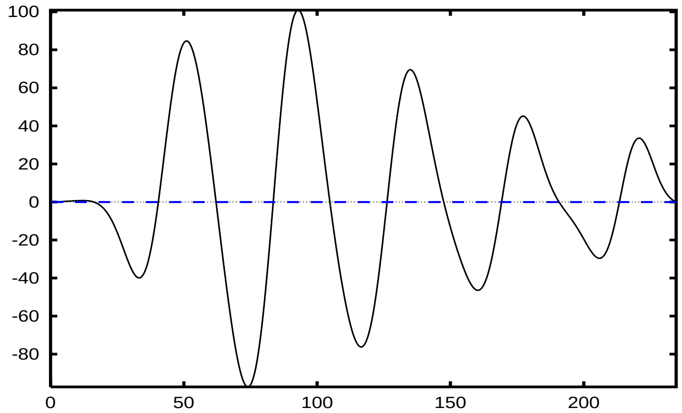
As MMO is approached new regions of advance and delay are added to the PRC

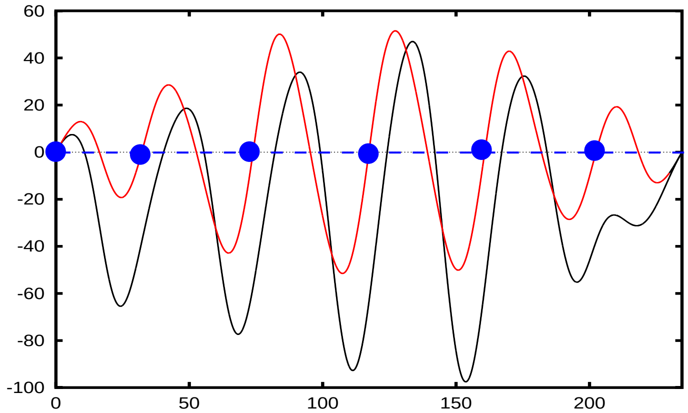
These build on the standard Hopf PRC

More complicated behavior

I=0.675

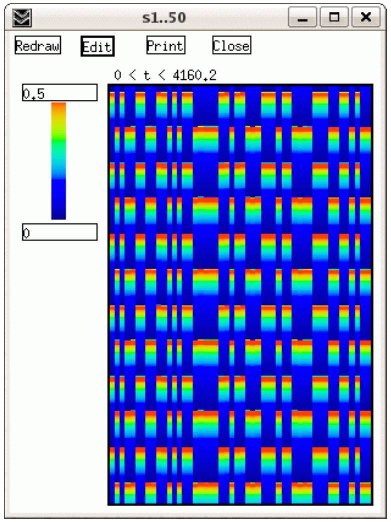






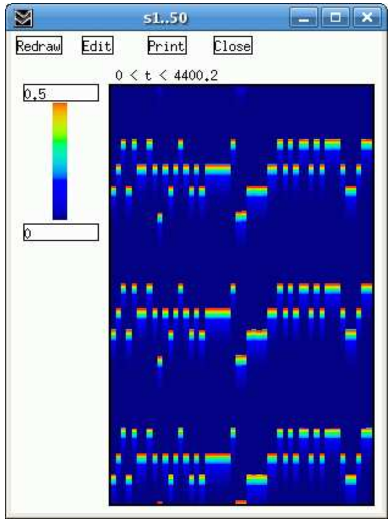
What about inhibitory coupling

I=0.7

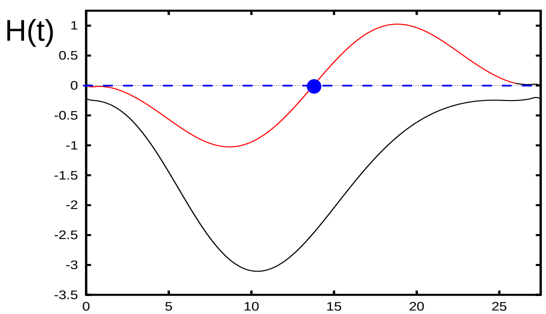


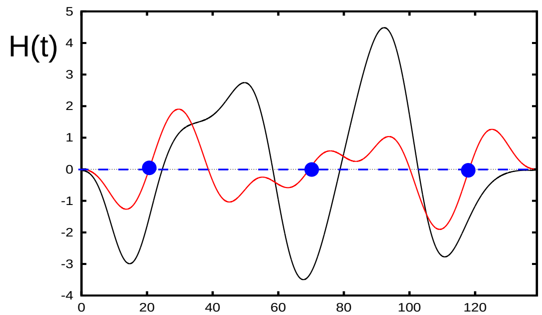
2 clusters

I=0.9



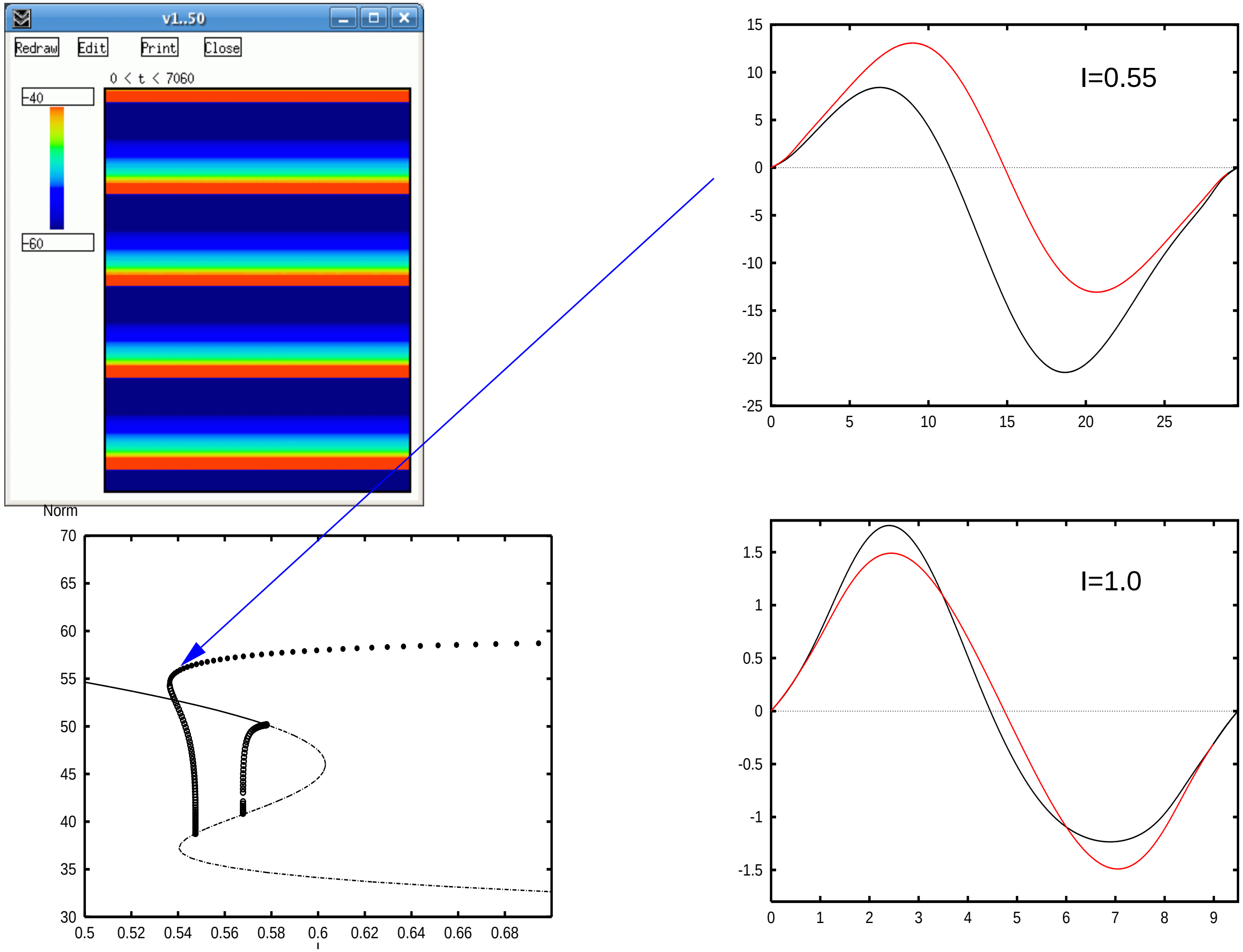
4 clusters



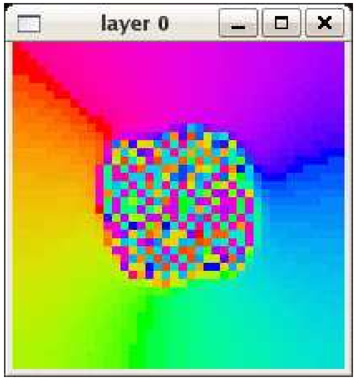


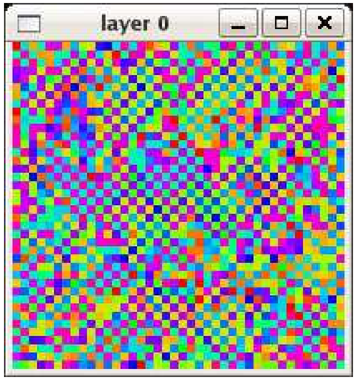
$E_{rev} = -70$ mV $\quad \tau = 8\,\mathrm{ms}$

Slow K is necessary



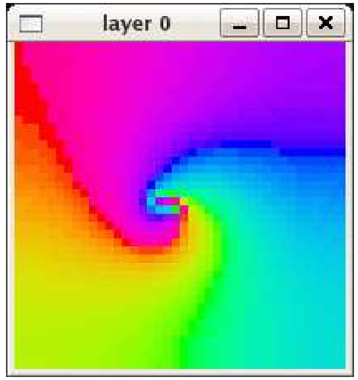
Two-d networks





I=0.75

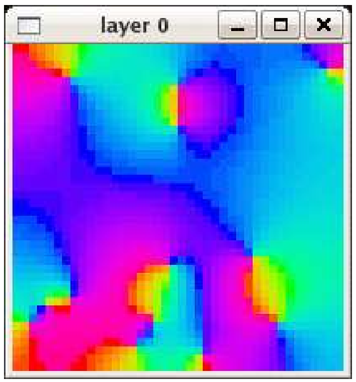
"Spiral ICs"



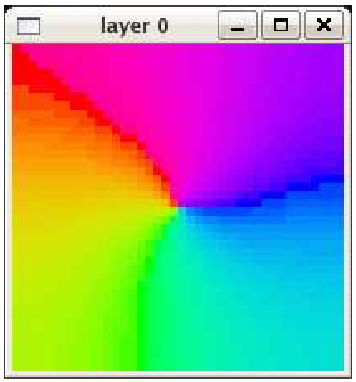
I=1.0

Spiral ICs

Random ICs



Spiral ICs



I=0.575 No GKS

In all 3 cases, synchrony

is a stable solution.

Caveats

- Coupling must be very weak
- MMO is extremely sensitive to perturbations
- Stronger gap junctions lead to synchrony
  - Larger perturbations reduce complexity of PRCs
  - Leads to a simplification of effective interaction function
- Similar results on coupled elliptic bursters